A novel SINR and mutual information based radar jamming technique

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Abstract: The improvements of anti-jamming performance of modern radar seeker are great threats to military targets. To protect the target from detection and estimation, the novel signal-to-interference-plus-noise ratio (SINR)-based and mutual information (MI) based jamming design techniques were proposed. To interfere with the target detection, the jamming was designed to minimize the SINR of the radar seeker. To impair the estimation performance, the mutual information between the radar echo and the random target impulse response was used as the criterion. The spectral of optimal jamming under the two criteria were achieved with the power constraints. Simulation results show the effectiveness of the jamming techniques. SINR and MI of the SINR-based jamming, the MI-based jamming as well as the predefined jamming under the same power constraints were compared. Furthermore, the probability of detection and minimum mean-square error (MMSE) were also utilized to validate the jamming performance. Under the jamming power constraint of 1 W, the relative decrease of the probability of detection using SINR-based optimal jamming is about 47%, and the relative increase of MMSE using MI-based optimal jamming is about 8%. Besides, two useful jamming design principles are concluded which can be used in limited jamming power situations.

Key words: detection; jamming; mutual information (MI); parameter estimation; minimum mean-square error (MMSE); probability of detection; signal-to-interference-plus-noise ratio (SINR)

1 Introduction

The electromagnetic environment of modern warfare is becoming increasingly complex. In order to shoot precisely, the anti-jamming performance of the radar seeker on the missile is greatly improved by using advanced signal processing methods. The enhancement of the detection probability, high resolution property of traditional radars and imaging radars, precision of target tracking, along with great identification performance are huge threats to the target. Figure 1 briefly shows the electronic warfare of the radar seeker and the target. For the sake of self-protection, many researchers pay attention to various effective jamming techniques, including active jamming and passive jamming, blanking and deceptive jamming, and so forth. The object of this work is to analyze the characteristics of optimal jamming, and acquire some guidance of jamming design techniques in limited power environments. The main contribution of this work is that owing to the usage of prior knowledge of the waveform, the target and environment noise characteristics, the proposed jamming design techniques can greatly reduce the detection and parameter estimation performance of the radar seeker using small jamming power compared with the widely used jamming techniques in electronic countermeasures (ECM). Thus, it is significant in the energy (power) limited environments which can achieve similar effect using smaller jamming power.

The proposed jamming design techniques in this work use signal-to-interference-plus-noise ratio (SINR)
and mutual information (MI) as the criteria. The optimal jamming is the jamming that can significantly reduce the SINR and MI of the radar seeker. SINR is an important factor for the radar seeker to detect the target, while MI is a measurement of information content which is related with target tracking or parameter estimation. In the perspective of the radar seeker (or general radar systems), a larger SINR indicates better detection probability. As a result, many investigations are done on improving SINR or signal-to-noise ratio (SNR) of the radar systems [1–3]. MI is a concept in information theory [4] that has been widely used in radar and other fields [5–6]. Information theory was first introduced in radar by WOODWARD [7–9]. Many radar engineers focus on improving MI to obtain better parameter estimation performance [10–14] and the relationship of MI and minimum mean-square error (MMSE) is discussed in Refs. [14–16] which show that maximizing mutual information will lead to more accurate estimation under some assumptions such as white noise and perfectly known noise PSD. As radar and jammer are considered as opponents in the battlefield, what should the jammer do if the radar system wants to have larger SINR and MI? A direct way is designing an optimal jamming which can reduce the SINR and MI of radar under some physical constraints of the jammer.

The scenario throughout this work is that the radar seeker is transmitting a waveform to detect, track or identify the target, while we are going to protect the target by jamming under the assumption that the waveform has already been intercepted. This is a nonspecific scenario that most of the jamming scenarios belong to. Apparently, if we can make deleterious effect to the detection, tracking and identification performance of the radar seeker, the target will be well protected. Based on the above analyses, two jamming techniques are proposed. The first optimal jamming minimizes the SINR of the radar seeker when the target is supposed to be known with a given impulse response. The minimal SINR will lead to poor detection performance of the radar seeker. The other minimizes the MI between the radar echo and the target impulse response when the target is supposed to be stochastic with a known power spectral density (PSD) $S_{\text{cc}}(f)$, which is non-zero over the entire waveform bandwidth. $e(t)$ denotes a complex-valued, zero-mean Gaussian random process representing an interference component and characterized by the PSD $S_{\text{in}}(f)$. The variables in bold denote the random process while others are deterministic.

2 Known-target signal model and SINR-based jamming design

As the larger the SINR is, the better the detection performance will be, and vice versa, the SINR-based jamming design technique is proposed to minimize the SINR of the radar seeker which leads to poor detection performance. In this section, the target is supposed to be known with a given target impulse response. The jammer intercepts the signal of the radar seeker, and acquires the waveform parameters as well as the orientation. Therefore, the target impulse response of the certain angle can be obtained by measurement in advance owing to the cooperativeness of the target. Figure 2 depicts the known target signal model [2]. The radar seeker transmits a waveform $x(t)$ to detect the target $h(t)$, which tells us the target signal model and SINR-based jamming design.

**Fig. 2 Known target signal model for SINR-based jamming design**

The main focus of this work is to find out the characteristics of $e(t)$ so that the performance of the radar seeker is weakened. Specifically speaking, we are going to design the PSD $S_{\text{cc}}(f)$, which will inspire us to design the active jamming signal with the power $J(f)=S_{\text{cc}}(f)X(f)^2$.

$$R_\text{at} = \frac{\int_{-\infty}^{\infty} R(f)H(f)X(f)e^{j2\pi f} df}{\int_{-\infty}^{\infty} [R(f)]^2 [X(f)]^2 S_{\text{cc}}(f) + S_{\text{in}}(f) df}$$

Applying the Schwartz’s inequality, the SINR achieves its maximum of

$$R_\text{at} = \frac{\int_{-\infty}^{\infty} [H(f)X(f)]^2 df}{\int_{-\infty}^{\infty} [X(f)]^2 S_{\text{cc}}(f) + S_{\text{in}}(f) df}$$

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if and only if the matched filter is of the form
\[
R(f) = \left[ xH(f)X(f)e^{j2\pi f_0} \right] \left[ X(f) \right]^2 S_{sc}(f) + S_{sn}(f) 
\]
(3)

Suppose that the transmitted signal and the jamming PSD are essentially limited to the bandwidth, \( W \), the maximum SINR may be written as
\[
R_s \approx \int \frac{[H(f)X(f)]^2}{[X(f)]^2 S_{sc}(f) + S_{sn}(f)} df 
\]
(4)
where \( R \) is related with the transmitted waveform, the target frequency response, noise PSD and the jamming.

The total power constraint of the jamming is supposed to be \( P \). Therefore, the optimization problem is
\[
\min R(S_{sc}(f)) \quad \text{s.t.} \quad \int \frac{S_{sc}(f)df}{W} \leq P 
\]
(5)

According to Theorem 1 in Appendix I, the solution of the problem is
\[
S_{sc}(f) = \max \left[ 0, B(f)(A-D(f)) \right] 
\]
(7)
where \( B(f) \) and \( D(f) \) are
\[
B(f) = |H(f)| \quad \text{and} \quad D(f) = \frac{S_{sn}(f)}{[X(f)]^2 |H(f)|} 
\]
(8)
(9)
respectively, and \( A \) is a constant determined by the power constraint:
\[
\int \max \left[ 0, B(f)(A-D(f)) \right] df \leq P 
\]
(10)
The optimal jamming that minimizes SINR has a frequency spectrum obtained by performing the waterfilling operation [4] on the function \( B(f)(A-D(f)) \). Note that this frequency-domain waterfilling has a fundamental limitation that the resulting jamming in time-domain is not time limited [2]. Note that if \( B(f) = |H(f)|^{-1} \), i.e., we do not have the prior knowledge of the target impulse response, so the target is supposed to be a point target with delta impulse response. Thus, the optimal PSD is only related with \( D(f) = S_{sn}(f)/[X(f)]^2 \). For the case where \( H(f) = 1 \), \( B(f) \) is the target-dependent factor that modifies the waterfilling operation by weighting the frequency component in \( A-D(f) \).

3 Finite-duration stochastic target and MI-based jamming design

In this section, the target is modeled as a random process which is stationary within the interval \([0, T_s]\) but is zero outside this interval [2, 10–11]. Figure 3(a) shows the creation of such a process. \( g(t) \) is a complex wide-sense stationary process with some PSD and \( a(t) \) is a rectangular window of duration \( T_s \). The product \( h(t) = a(t)g(t) \) is a finite-duration random process having support only in \([0, T_s]\). \( h(t) \) is locally stationary within \([0, T_s]\) since \( g(t) \) is wide-sense stationary.

**Fig. 3** Random target signal model for MI-based jamming design [2]: (a) Signal model for finite-duration random target; (b) Signal model for MI-based jamming design

The energy spectral variance (ESV) of \( h(t) \) can be defined as [2, 10–11]
\[
\sigma_H^2(f) = E\left[ |H(f) - \mu_H(f)|^2 \right] 
\]
(11)
where \( E[\cdot] \) means expectation, \( H(f) \) is the Fourier transform counterpart of \( h(t) \), and \( \mu_H(f) \) is the mean of \( H(f) \), i.e., \( \mu_H(f) = E[H(f)] \). The ESV function describes the average energy of a finite-duration, zero-mean process in the same sense that a PSD describes the average power of an infinite-duration, wide-sense stationary process. Assuming that \( \mu_H(f) = 0 \), we define
\[
\gamma_H(f) = \frac{\sigma_H^2(f)}{T_s} = \frac{E\left[ |H(f)|^2 \right]}{T_s} 
\]
(12)
as the power spectral variance (PSV).

Consider again the signal model in Fig. 3(b). The approximate mutual information between the echo and the random target impulse response is [2]
\[
I = I (\gamma(t); h(t)) = T_s \int \ln \left[ 1 + \frac{(X(f))^2 \gamma_H(f)}{S_{sn}(f) + (X(f))^2 S_{sc}(f)} \right] df 
\]
(13)
where \( \alpha = T_0/T_s \), with \( T_0 \) denoting the duration of the echo \( y(t) \). MI is related with the transmitted waveform, the target PSV, the noise PSD and the jamming.

The optimal jamming should satisfy
\[
\min I(S_{cc}(f))
\]

s.t.
\[
\int_{w} S_{cc}(f) df \leq P
\]

The minimization of the mutual information between the echo and the random target impulse response implies that the echo contains as little information of the target as possible, which will lead to poor tracking or parameter estimation performance of the target.

The optimal jamming that minimizes the MI is
\[
\hat{S}_{cc}(f) = \max \left[ 0, B(f)[A + D(f)] \right]
\]

where
\[
B(f) = -\frac{\alpha \gamma_H(f) |X(f)|^2}{2S_{nn}(f) + \alpha \gamma_H(f) |X(f)|^2}
\]

and
\[
D(f) = \frac{S^2_{nn}(f) + \alpha \gamma_H(f) |X(f)|^2 S_{nn}(f)}{\alpha \gamma_H(f) |X(f)|^4}
\]

respectively. \(A\) is a constant which is calculated according to the power constraint of the jamming:
\[
\int_{w} \max \left[ 0, B(f)(A + D(f)) \right] df \leq P
\]

The derivation is shown in Theorem 2 in Appendix II. Note that if \(\alpha \gamma_H(f)\) is relatively large, then \(B(f) \rightarrow -1, D(f) \rightarrow S_{nn}(f) |X(f)|^2\), where \(\rightarrow\) means “coming close to”. The resulted forms of the MI-based jamming and the SNR-based jamming under the condition that \(|H(f)|=1\) are the same if we put the minus of \(B(f)\) into the bracket and replace \(-A\) by \(A\) of Eq. (19). This describes the relationship of the SINR-based jamming and the MI-based jamming.

4 Simulations

The simulations were implemented in MATLAB on an Intel Pentium 3 GHz Dual-Core CPU, and 2 GB RAM computer. The whole procedure of the proposed two methods is shown in Fig. 4. In the main procedure (Fig. 4(a)), the prior knowledge of the transmit waveform, the target frequency response (known target signal model) or the target PSV (stochastic target model) together with the noise PSD is given. Then, for the two criteria, the total jamming power is given. After using the prior knowledge to obtain the variable \(B(f)\) and \(D(f)\), the waterfilling method is applied to calculate \(A\). Once \(A\) is acquired, the optimal interference PSD \(S_{cc}(f)\) or jamming spectrum \(J(f) = S_{cc}(f) |X(f)|^2\) as well as the minimum value of SINR and MI are easily got. Waterfilling procedure is the key process in the two methods. As the integral in Eqs. (10) and (19) are all monotone functions of \(A\), the binary search algorithm can be utilized to find the most approximate \(A\) which has the error of less than a given upper bound \(\varepsilon\). Figure 4(b) shows the waterfilling procedure, where the inputs \(a\) and \(b\) satisfy \(S(A=a) < P\) and \(S(A=b) > P\), respectively. And \(S(A)\) is calculated using the left side integral in Eqs. (10) and (19) under SINR criterion and MI criterion, respectively.

Given the Fourier transform of the waveform \(X(f)\), the noise PSD \(S_{nn}(f)\), the target frequency impulse response \(H(f)\) for known target, and the target PSV \(\gamma_H(f)\) for stochastic target, we use the aforementioned SINR-based jamming design method and the MI-based jamming design method to find out the optimal jamming.

![Fig. 4 Flow chart of proposed method: (a) Main procedure; (b) Waterfilling procedure](image-url)
under the maximum power constraint. In Fig. 5, the Fourier transform of the transmit waveform is represented with a total energy of 1 J and the main energy integrated near the normalized frequency of 0.3. The target frequency response (known target model) is illustrated with the total energy of 1 J and the main energy integrated near the normalized frequency of −0.2, 0, and 0.4, respectively. To make similar environment for the stochastic target model, the stochastic target ESV is supposed to be \( \sigma_{H}^{2}(f) = |H(f)|^{2} \) (expectation operator is omitted so this is a realization of the random variable), where \( H(f) \) is the given target frequency response for known target model. Therefore, \( \sigma_{H}^{2}(f) \) is not illustrated in Fig. 5 because it has the similar shape with \( H(f) \). The duration of the realization of the echo \( y(t) \) is assumed to be \( T_{f}=1.5 \times 10^{-3} \) s. Therefore, \( \sigma_{y}^{2}(f) \) is obtained. The noise PSD is set so that the total noise power is 1 W.

Figure 5 also illustrates the three jamming, including the SINR-based jamming spectrum, the MI-based jamming spectrum along with the predefined jamming spectrum. In Fig. 5(a), the jamming power constraint is \( P=1 \) W while in Fig. 5(b) \( P=5 \) W. The predefined jamming is supposed to be a barrage noise jamming with a flat PSD in the frequency band of the waveform, which is widely used in jamming method recently. Both the optimal SINR-based jamming and the MI-based jamming place their limited power into the frequency band of the waveform, which proves the effectiveness of the widely used barrage jamming method that spread its power into the radar frequency bands (such as the predefined jamming). Furthermore, the SINR-based optimal jamming concentrates its power not only into the frequency band of the waveform, but also into the frequency band of the target. Not similarly, the MI-based jamming only spread its power into the frequency band of the waveform. With the increase of the jamming power constraint, the MI-based jamming still keeps the property of placing most of its power into the frequency band of the waveform, with a little spread of the jamming bandwidth (In fact, with proper power, the MI-based jamming tends to allocate its power into the whole frequency band). While the optimal SINR-based jamming shows more explicit property of allocating its power into the frequency band of both the waveform and the target. The results give us some inspiration when creating jamming.

Suppose that the power constraint of the jamming changes from 1 W to 10 W, the SINR and the MI of the radar seeker when using the SINR-based optimal jamming, the MI-based optimal jamming and the predefined jamming are compared in Fig. 6. In Fig. 6(a), the SINR-based optimal jamming achieves the lowest SINR as we have expected, which shows the effectiveness of the SINR-based jamming design method. Owing to the low SINR, the jamming will have good performance in preventing the radar from detecting the target. The SINR of the MI-based optimal jamming is a little larger, while the predefined jamming leads to the largest SINR. This is owing to the fact that the predefined jamming utilizes the frequency band information of the transmit waveform only, while the MI-based jamming utilizes more information. For instance, when jamming power constraint is 10 W, SINR of the SINR-based optimal jamming is about −12.5 dB, while that of the predefined jamming is about −8.7 dB, i.e., the proposed SINR-based jamming technique can result in a SINR reduction of about 3.8 dB using the same jamming power. In other words, to achieve the same jamming effect (same SINR), using the proposed SINR-based jamming technique can save more power than using the predefined jamming technique. For example, to get a SINR of −8 dB, the jamming power used by SINR-based jamming technique is about 2 W, while that by the predefined jamming technique is about
6 W. This is significant in the resource constraint and power limited environment. Besides, the SINR of using the SINR-based optimal jamming decreases more sharply than that of the other two with the jamming power increasing, which can also prove the effectiveness.

Figure 6(b) illustrates the MI of the three jamming. The best jamming in the sense of MI is the MI-based optimal jamming, while the SINR-based optimal jamming results in a larger MI, and the predefined jamming has the largest MI. It can also be concluded that using the same jamming power, the proposed MI-based jamming technique can result in better jamming performance with the lowest MI; besides, to achieve the same MI, the proposed jamming techniques can save much power. For example, when jamming power constraint is 7 W, the MI of the MI-based jamming, SINR-based jamming and predefined jamming is 0.005 5 nats, 0.005 7 nats, and 0.005 8 nats, respectively. Although the absolute decrease is slight (10^{-4} order of magnitude), the relative decrease is significant because MI is very small (10^{-3} order of magnitude).

As SINR and MI are related with the detection and parameter estimation performance [1−3, 10−14] respectively, the probability of detection and the MMSE of estimating the target impulse response are calculated to validate the effectiveness of the jamming techniques. The probability of false alarm is set to be 10^{-3}, and the probability of detection is calculated according to Appendix III once the jamming spectral and other related signals are obtained. MMSE can be calculated using Appendix IV. Results are shown in Fig. 7. It is manifest that the probability of detection is greatly reduced using the SINR-based jamming technique compared with that using the predefined jamming. The MMSE of the MI-based jamming is larger than that of the predefined jamming, which indicates that large estimation error will occur using the designed jamming. Therefore, the jamming technique greatly reduces the efficiency of the radar seeker using the least jamming power. Note that the performance enhancement of the designed jamming with respect to the predefined jamming is the main focus in this work, and the given powers in the scenario are not typical values in practice, and the values of SINR and MI are very small under the designed scenario, which leads to small probability of detection and large MMSE.

To reveal the performance improvement more explicitly, Fig. 8(a) illustrates the relative decrease of probability of detection, which is calculated by
\[
\left(\frac{P_D^{\text{predefined}} - P_D^{\text{SINR-based}}}{P_D^{\text{predefined}}}\right) \times 100\% ,
\]
where \( P_D^{\text{predefined}} \) is the probability of detection of the predefined jamming.
jamming and $p_{D}^{\text{SINR-based}}$ is that of SINR-based optimal jamming. Figure 8(b) shows the relative increase of MMSE which is calculated similarly. If the jamming power is 1 W, the relative decrease of probability of detection is about 47%, and the relative increase of MMSE is about 8%. Both of them will lead to evident effect in jamming the radar seeker. Obviously, the smaller the jamming power constraint is, the more improvement the two jamming techniques will obtain. Thus, the proposed two jamming techniques are significant in power limited environment.

Based on the aforementioned simulations and analyses, the following two principles of jamming design are concluded.

**Principle 1:** To prevent the target from detection of the radar seeker, the optimal jamming follows the minimum SINR criterion, which allocates its finite power into the frequency band of the waveform. With the increase of the jamming power, the optimal MI jamming tends to spread its spectrum into larger frequency bands including the frequency band of the target.

**5 Conclusions**

1) The proposed SINR-based and MI-based jamming design techniques are effective which will reduce the SINR and MI of the radar seeker, respectively. Thus, the detection performance and the parameter estimation performance of the radar seeker are interfered, through a decrease of probability of target detection and an increase of the MMSE when estimating the target impulse response compared with the widely used barrage noise jamming. Through the analyses of the relative performance improvement, it can be concluded that the two jamming design techniques are more significant in limited power environment.

2) Both the two jamming design methods give us guidance of jamming design. Firstly, the optimal jamming in SINR sense will place its power into the frequency band of the waveform and the target impulse response. Secondly, the optimal jamming in MI sense will only place its power into the frequency band of the waveform. However, when the jamming power increases, the MI-based optimal jamming will spread its energy into larger frequency bands.

3) For practical use, there are still other problems that should be concerned, such as the change of the optimal jamming when the transmit waveform, the target orientation and the environment of noise change. And the realization of the optimal jamming should be studied in future work. Besides, a smart jammer which can adaptively interfere with the radar seeker and adjust its criterion according to the different stages of the radar seeker will be the ultimate goal.

**Appendix I: Derivation of SINR-based optimal jamming**

**Theorem 1:** $S_{cc}(f)$ that minimizes SINR equation given by

$$R_{u} \simeq \int_{W} \frac{|H(f)X(f)|^{2}}{|X(f)|^{2}S_{cc}(f) + S_{m}(f)} df$$

(20)
given the constraint

$$P \geq \int_{W} S_{cc}(f) df$$

(21)
is

$$S_{cc}(f) = \max \left[ 0, B(f)(A - D(f)) \right]$$

(22)

where
$B(f) = |H(f)|$  \hspace{1cm} (23) \\
$D(f) = \frac{S_{\text{sn}}(f)}{|X(f)|^2 |H(f)|}$  \hspace{1cm} (24)

**Proof:** We invoke the Lagrangian multiplier technique yielding an objective function:

\[
K(S_{\text{cc}}(f), \lambda) = \int_w \frac{|H(f)|^2 |X(f)|^2}{S_{\text{cc}}(f)|X(f)|^2 + S_{\text{sn}}(f)} df + \lambda \left[ P - \int_w S_{\text{cc}}(f) df \right]  \hspace{1cm} (25)
\]

This is equivalent to minimizing $k(S_{\text{cc}}(f))$ with respect to $S_{\text{cc}}(f)$ where $k(S_{\text{cc}}(f))$ is given by

\[
k(S_{\text{cc}}(f)) = \frac{|H(f)|^2 |X(f)|^2}{S_{\text{cc}}(f)|X(f)|^2 + S_{\text{sn}}(f)} - \lambda S_{\text{cc}}(f)  \hspace{1cm} (26)
\]

The second-order derivation of $k(S_{\text{cc}}(f))$ with respect to $S_{\text{cc}}(f)$ is greater than zero. Therefore, taking the derivation of $k(S_{\text{cc}}(f))$ with respect to $S_{\text{cc}}(f)$ and setting it to zeros yields the $S_{\text{cc}}(f)$ that minimizes Eq. (20), where $S_{\text{cc}}(f)$ is given by

\[
S_{\text{cc}}(f) = \frac{|H(f)|^2}{\sqrt{-\lambda}} \frac{S_{\text{sn}}(f)}{|X(f)|^2}  \hspace{1cm} (27)
\]

where $\lambda < 0$. Setting $A = 1/\sqrt{-\lambda}$ and ensuring $S_{\text{cc}}(f)$ to be positive, the $S_{\text{cc}}(f)$ that minimizes SINR is given by

\[
S_{\text{cc}}(f) = \max \left[ 0, \frac{|H(f)|}{|X(f)|^2} \left( A - \frac{S_{\text{sn}}(f)}{|X(f)|^2 |H(f)|} \right) \right]  \hspace{1cm} (28)
\]

**Appendix II:** Derivation of MI-based optimal jamming

**Theorem 2:** $S_{\text{cc}}(f)$ that minimizes MI equation given by

\[
I(y(t); h(t)) = T_{\text{y}} \int_w \ln \left[ 1 + \alpha \frac{|X(f)|^2 \gamma_H(f)}{S_{\text{sn}}(f) + |X(f)|^2 S_{\text{cc}}(f)} \right] df  \hspace{1cm} (29)
\]

given the constraint

\[
P \geq \int_w S_{\text{cc}}(f) df \hspace{1cm} (30)
\]

is given by

\[
\tilde{S}_{\text{cc}}(f) = \max \left[ 0, B(f) \left[ A + D(f) \right] \right]  \hspace{1cm} (31)
\]

where

\[
B(f) = -\frac{\alpha \gamma_H(f) |X(f)|^2}{2S_{\text{sn}}(f) + \alpha \gamma_H(f) |X(f)|^2}  \hspace{1cm} (32)
\]

\[
D(f) = \frac{S_{\text{sn}}(f)}{\alpha \gamma_H(f) |X(f)|^2} \left[ \frac{S_{\text{sn}}(f) + \alpha \gamma_H(f) |X(f)|^2}{2S_{\text{sn}}(f) + \alpha \gamma_H(f) |X(f)|^2} \right]  \hspace{1cm} (33)
\]

**Proof:** We invoke the Lagrangian multiplier technique yielding an objective function:

\[
K(S_{\text{cc}}(f), \lambda) = \int_w \ln \left[ 1 + \alpha \frac{|X(f)|^2 \gamma_H(f)}{S_{\text{sn}}(f) + |X(f)|^2 S_{\text{cc}}(f)} \right] df + \lambda \left[ P - \int_w S_{\text{cc}}(f) df \right]  \hspace{1cm} (34)
\]

This is equivalent to minimizing $k(S_{\text{cc}}(f))$ with respect to $S_{\text{cc}}(f)$ where $k(S_{\text{cc}}(f))$ is given by

\[
k(S_{\text{cc}}(f)) = T_{\text{y}} \int_w \ln \left[ 1 + \frac{\alpha |X(f)|^2 \gamma_H(f)}{S_{\text{sn}}(f) + |X(f)|^2 S_{\text{cc}}(f)} \right] df - \lambda S_{\text{cc}}(f)  \hspace{1cm} (35)
\]

The second-order derivation of $k(S_{\text{cc}}(f))$ with respect to $S_{\text{cc}}(f)$ is greater than zero. Therefore, taking the derivation of $k(S_{\text{cc}}(f))$ with respect to $S_{\text{cc}}(f)$ and setting it to be zero yields the $S_{\text{cc}}(f)$ that minimizes Eq. (29).

The optimal $S_{\text{cc}}(f)$ is

\[
S_{\text{cc}}(f) = \max \left[ 0, -R(f) + \sqrt{R^2(f) - S(f)[A + D(f)]} \right]  \hspace{1cm} (36)
\]

where $A$ is a constant determined by the power constraint

\[
\int_w \max \left[ 0, -R(f) + \sqrt{R^2(f) - S(f)[A + D(f)]} \right] df \leq P  \hspace{1cm} (37)
\]

and

\[
R(f) = \frac{S_{\text{sn}}(f)}{|X(f)|^2} + \frac{\alpha \gamma_H(f)}{2}  \hspace{1cm} (38)
\]

\[
S(f) = \alpha \gamma_H(f)  \hspace{1cm} (39)
\]

\[
D(f) = \frac{S_{\text{sn}}^2(f) + \alpha \gamma_H(f) |X(f)|^2 S_{\text{sn}}(f)}{\alpha \gamma_H(f) |X(f)|^2}  \hspace{1cm} (40)
\]

respectively.

We apply first-order Taylor approximation to

\[
Q(f) = -R(f) + \sqrt{R^2(f) - S(f)[A + D(f)]}  \hspace{1cm} (41)
\]

and get

\[
\tilde{Q}(f) = B(f) \left[ A + D(f) \right]  \hspace{1cm} (42)
\]

where

\[
B(f) = -\frac{\alpha \gamma_H(f) |X(f)|^2}{2S_{\text{sn}}(f) + \alpha \gamma_H(f) |X(f)|^2}  \hspace{1cm} (43)
\]

Therefore,

\[
\tilde{S}_{\text{cc}}(f) = \max \left[ 0, B(f) \left[ A + D(f) \right] \right]  \hspace{1cm} (44)
\]
Appendix III: Derivation of detection performance

For the known-target signal model in Section 2, the probability of detection is derived to validate the performance of the proposed SINR-based jamming technique.

Suppose that the jamming PSD $S_{vc}(f)$ is obtained using the proposed SINR-based jamming technique. The receiver output of the radar seeker is

$$y(t) = r(t) * [x(t) * h(t) + x(t) * c(t) + n(t)] = r(t) * [s(t) + \omega(t)]$$

(45)

where the noise and jamming are represented together by $\omega(t)$, and $s(t)=x(t)*h(t)$. Let

$$y = \left[ y(1) \ y(2) \ \cdots \ y(L) \right]^T$$

$$s = \left[ s(1) \ s(2) \ \cdots \ s(L) \right]^T$$

$$\omega = \left[ \omega(1) \ \omega(2) \ \cdots \ \omega(L) \right]^T$$

(46)

(47)

(48)

and the receiver convolution matrix be

$$R = \begin{bmatrix}
    r(1) & 0 & \cdots & 0 \\
    r(2) & r(1) & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    r(L) & r(L-1) & \cdots & r(1) \\
    r(L+1) & r(L) & \cdots & r(2) \\
    \vdots & \vdots & \ddots & \vdots \\
    r(L) & r(L-1) & \cdots & r(L-L) + 1 \\
    0 & r(L) & \cdots & r(L-L) + 2 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & r(L) \\
\end{bmatrix}_{L_x \times L_x}$$

(49)

where $L_x = L + L - 1$. Therefore, the receiver output can be shown in vector form

$$y = Rs + R\omega$$

(50)

Let $s' = Rs$, $\omega' = R\omega$, the detection problem can be represented by a binary hypothesis test:

$$H_0 : y = \omega'$$

$$H_1 : y = s' + \omega'$$

(51)

where $\omega'$ is a zero mean Gaussian random vector with the PSD

$$S_{\omega'}(f) = \left| R(f) \right|^2 \left( S_{cc}(f) |X(f)|^2 + S_{nn}(f) \right)$$

$$\left| H(f)X(f) \right|^2 \left| X(f) \right|^2 S_{cc}(f) + S_{nn}(f)$$

(52)

It is apparent that the PSD is the integrand of SINR in Eq. (2). Applying the Wiener-Khinchine theorem, the covariance matrix of $\omega'$, denoted by $\Sigma$, can be obtained, which is related with SINR.

Using the Neyman-Pearson detector, the test statistic is $T(y) = y^T \Sigma^{-1} s'$, which is a typical mean-shifted Gauss-Gauss problem.

$$T(y) \sim \begin{cases} 
    N \left( 0, s'^T \Sigma^{-1} s' \right) \\
    N \left( s'^T \Sigma^{-1} s', s'^T \Sigma^{-1} s' \right)
\end{cases}$$

(53)

The probability of detection is [17]

$$P_D = Q \left( Q^{-1}(P_{FA}) - \sqrt{s'^T \Sigma^{-1} s'} \right)$$

(54)

where $Q(x)$ is defined as

$$Q(x) = \int_{x}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

(55)

Given the probability of false alarm $P_{FA}$ together with $X(f)$, $H(f)$, $S_{vc}(f)$, and the three jamming PSDs, the probability of detection is easily obtained. The term $s'^T \Sigma^{-1} s'$ is related with SINR. Larger SINR results in larger probability of detection.

Appendix IV: Derivation of minimum mean-square error

Using the random target model in Section 3, the minimum mean-square error (MMSE) of the estimation problem can be derived:

$$y(t) = x(t) * h(t) + \omega(t)$$

(56)

where the noise and jamming are represented together by $\omega(t)$. Let $h = [h(1) \ h(2) \ \cdots \ h(L_h)]^T$ be a Gaussian random target impulse response with zero mean and covariance matrix $\Sigma_h = E[hh^H]$. The waveform convolution matrix is

$$X = \begin{bmatrix}
    x(1) & 0 & \cdots & 0 \\
    x(2) & x(1) & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    x(L_h) & x(L_h-1) & \cdots & x(1) \\
    x(L_h+1) & x(L_h) & \cdots & x(2) \\
    \vdots & \vdots & \ddots & \vdots \\
    x(L) & x(L-1) & \cdots & x(L_h-1) \\
    0 & x(L) & \cdots & x(L_h) \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & x(L) \\
\end{bmatrix}_{L_x \times L_h}$$

(57)

where $L_x = L + L - 1$. Let

$$\omega = \left[ \omega(1) \ \omega(2) \ \cdots \ \omega(L) \right]^T$$

(58)
\[ y = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(L_y) \end{bmatrix} \]  \tag{59}

where \( \omega \) is a zero mean colored Gaussian noise with PSD \( S_{\omega \omega}(f) = S_{\omega m}(f) + S_{\omega c}(f) |X(f)|^2 \). Thus its covariance matrix \( \Sigma_{\omega} \) can be easily got. The signal model can be represented in vector form:

\[ y = Xh + \omega \]  \tag{60}

The error covariance matrix of the linear MMSE estimator is \[[18]\]

\[ \Sigma_{e} = (\Sigma_{h}^{-1} + X^{H} \Sigma_{\omega}^{-1} X)^{-1} \]  \tag{61}

MMSE can be obtained by

\[ \text{MMSE} = \text{tr}\{(\Sigma_{h}^{-1} + X^{H} \Sigma_{\omega}^{-1} X)^{-1}\} \]  \tag{62}

References


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