Design and analysis of logarithmic spiral type sprag one-way clutch

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Abstract: A complete mathematical model for logarithmic spiral type sprag one-way clutch design and analysis is given. It assumes that the motion of all clutch components can be expressed by a model of epicyclic gearing. It takes advantage of Hunt-Crossley contact impact theory to calculate the contact forces between sprags and races, and it can be used for optimization of design and comparison with other types of sprag clutches. A good deal of analysis shows that the parameters of the steady windup angle, the steady contact force, the natural frequency and natural cycle of clutch have nothing to do with the initial velocity of outer race, while the parameters of the maximum transient windup angle, the maximum transient impact force and the steady engagement time increase linearly in the mode of engaging operation of clutch. It is also shown that the strut angle has great influence on the dynamic engagement performance of clutch. The parameters of the steady windup angle, the maximum transient windup angle, the steady engaging time, the steady contact force, the maximum transient impact force and the natural cycle of clutch decrease linearly nearly with the inner strut angle, while the natural frequency of the system increases linearly with the inner strut angle.

Key words: mathematical model; logarithmic spiral; one-way clutch; hunt-crossley; windup angle; steady contact force; transient impact force

1 Introduction

There are presently two major types of one-way clutches (OWC): roller and sprag in the helicopter transmission system. The sprag OWC is widely used because its manufacturing cost, centrifugal force effect and the reliability are superior to the roller OWC. And there are three main kinds of working surface in the cam profile of sprag OWC including circular arc type profile, logarithmic spiral type profile and Archimedes spiral type profile.

There are existing studies on the profile design of sprag cam and dynamics analysis of sprag OWC. HU [1] calculated the strut angle of the logarithmic spiral type of roller OWC, the cam of which is commonly integrated with outer race, and discussed the load capacity of logarithmic spiral type sprag OWC. ZHU [2] designed the eccentric circular arc type sprag OWC and tested its performance by means of experimentation. HU [3] studied the function of strut angle of the eccentric circular arc type sprag OWC and analyzed the contact force between the sprags and the races. HE et al [4] discussed the method to the design of eccentric circular arc cam profile of roller clutch. LIU and ZHENG [5] finished the three-dimensional design of logarithmic spiral type sprag OWC and analyzed the contact stress and strain between the sprags and the races. WILLIAMS et al [6] analyzed the dynamic characteristics of sprag OWC and established a slide-model considering multiple factors during its engagement process. CRAMTON et al [7] tested the dynamic engagement characteristics of sprag OWC by means of experimentation. CHASSAPIS and LOWEN [8] established a nonlinear dynamic model for the analysis of the dynamic engagement characteristics of sprag OWC. YAN et al [9] analyzed the dynamic characteristics of the single circular arc type of sprag OWC and discussed the performance of positive continuous engagement (PCE) type sprag OWC under the condition of different frictions. WU and YAN [10–11] designed the logarithm spiral PCE type sprag OWC and simulated its dynamic performance with the help of business analytic software.

As for the studies on the dynamic characteristics of logarithmic spiral type sprag OWC, the experts still haven’t been able to establish a nonlinear dynamic analysis model, because presently these researches have to rely on the simulation software such as ADAMS. At
broad, though XU and LOWEN [12], and CHANG [13] successively established a nonlinear dynamic mathematical model for circular arc sprag clutch and the roller clutch analysis, these models are not suitable for logarithmic spiral sprag clutch.

The primary contribution of this work is to develop a complete mathematical model to design and analyze logarithmic spiral type sprag OWC. The parametric design of logarithmic spiral type OWC is based on its geometric structure and working conditions. The whole development process of the mathematical model is given, which assumes the motion of all clutch components can be expressed by a model of epicyclic gearing, and takes advantage of the Hunt-Crossley contact impact theory to calculate the contact forces between the sprags and the races. It can be used for optimization design and comparison with other types of sprag clutches, and it provides a new method for designing logarithm spiral type sprag clutch with long life and good dynamic engagement properties.

2 Parametric design of logarithmic spiral type sprag OWC

The logarithmic spiral type sprag OWC is mainly comprised of inner race, circular array sprags, outer race, and spring [11]. The sprag is one of the key components of clutch. And whether the design of the sprag cam profile is reasonable or not is directly related to the performance and the service life of clutch.

Figure 1 gives a simplified schematic of the basic components and several variables. These variables are used throughout this work. Points \( P \) and \( Q \) are the contact points between the sprag and the outer race and the inner race, respectively. It is assumed that the center of circular arc type outer cam and the pole of the logarithmic spiral type inner cam are the same point \( O' \).

The inner strut angle \( \alpha_i \) is defined as the angle between the radial line \( OE \) drawn through the contact point \( Q \) and the strut line \( PQ \) drawn through the two contact points \( P \) and \( Q \). The outer strut angle \( \alpha_o \) is defined as the angle between the radial line \( OP \) drawn through the points \( O' \) and \( P \) and the strut line \( PQ \). The angle \( \gamma \) is the intersection angle between the polar radius \( QO' \) and the strut line \( PQ \). The angle \( \beta \) is the angle between the radial line \( OP \) and the polar radius \( OQ \) and the angle \( \psi \) is defined as the angle between two radial lines of \( OP \) and \( OQ \). The angle \( \Omega \) is called rotation angle, which is between the horizontal line \( OE \) and the line \( MN \). The size of rotation angle \( \Omega \) is changeable with the rotation of the sprag between two races. \( R_i \) is the radius of inner surface of outer race and \( R_o \) is the radius of outer surface of inner race.

2.1 Determination of strut angle

The OWC has two modes of operation. The first one is the engaged mode, and the second one is the freewheel mode [14]. When the sprag OWC is in the engaged mode of operation, the sprag is engaged between the inner race and the outer race, and it transmits torque load from inner race to outer race (or vice-versa). The contact force \( F_i \) between the sprag and the inner race can be decomposed into the normal contact force \( F_{ni \ } \) and the tangential contact force \( F_{fi \ } \). In order to reduce the phenomena of slipping and popping of sprag occurring, the relationship between \( F_{ni \ } \) and \( F_{fi \ } \) should be given as

\[
\tan \alpha_i = \frac{F_{ni}}{F_{fi}} \leq \mu_s \tag{1}
\]

Since the inner strut angle \( \alpha_i \) is always larger than the outer strut angle \( \alpha_o \), we only need to insure \( \alpha_i \) satisfying the conditions of self-locking. The coefficient of static friction \( \mu_s \) has a range of acceptable variation. For hardened steel on hardened steel, which is assumed to be bathed in automatic fluid lubrication, \( \mu_s \) is between
0.08 and 0.12. Taking the conservative value of 0.08, we can have the maximum inner race strut angle by Eq. (1) as
\[ \alpha_i \leq 4.574^\circ \]

The value of \( \alpha_i \) cannot be too small, because if the inner race strut angle is much smaller, the sprag has poorer ability to be disengaged. While the value of \( \alpha_i \) cannot be too large, because if the inner race strut angle is much larger, the sprag is much harder to be engaged between inner race and outer race. The ideal value of \( \alpha_i \) is 2°–4° [15].

2.2 Determination of initial rotation angle

The initial rotation angle \( \Omega \) mentioned above and shown in Fig. 2(a) is defined as the intersection angle between the horizontal axis of design coordinate of sprag and the horizontal direction line of clutch when in the freewheel mode of operation. The corresponding arc angle of outer cam is shown in Fig. 2(b). The range of rotation angle may be between \(-27.3^\circ\) and \(27.3^\circ\). During the operation of torque transmittal mode of sprag OWC, it is necessary to limit the system deflections. If the sprag rotates too far, due to the system deflections, the sprag will rollover and permanently wedge itself between the inner race and the outer race. And if the torque load applied on the inner race (or outer race) is much larger, the deflection will be much larger, and the angle that the sprag revolves around its own center will be much larger. In order to ensure that the sprag would not rollover, transmit this the maximum torque, we should choose the maximum value of the rotation angle \( \Omega \). Considering a certain margin of safety, the initial rotation angle of logarithmic spiral type sprag clutch can be chosen as 22°, which is not just the only value, but in a certain range.

![Diagram](image)

Fig. 2 Diagrammatic sketch of rolling angle: (a) Initial contact mode; (b) Range of arc angle of inner cam

2.3 Determination of parameters of logarithmic spiral equation

Considering the geometric relationships of each components of sprag OWC shown in Fig. 1, and the property of logarithmic spiral equation [11, 16], it is obtained:

\[ \alpha_i = -\arctan \frac{R_i \sin \psi}{R_o - R_i \cos \psi} \]  
\[ \psi = \arctan m - \arcsin \left( \frac{mR_o}{(R_i - r_o)\sqrt{1 + m^2}} \right) \]  
\[ \beta = \arcsin \left( \frac{mR_o}{(R_i - r_o)\sqrt{1 + m^2}} \right) \]  
\[ \rho = \frac{R_o \sin \psi}{\sin \beta} \]  
\[ a = \frac{\rho}{e^{m\psi}} \]  
\[ \Omega = 1.5\pi + \beta - \varphi \]

From Eq. (3) to Eq. (8), we know that if we have the value of the rotation angle \( \Omega \), the inner strut angle \( \alpha_i \), the radius of inner surface of outer race \( R_i \) and the radius of outer surface of inner race \( R_o \), we can calculate the value of the parameters \((m, a)\) in the logarithmic spiral equation.

3 Dynamic analysis of logarithmic spiral type sprag clutch

The OWC usually has several working modes including the initial contact, dynamic engaging, torque transmitting, overload and reverse overrunning [18]. The modes of dynamic engaging, torque transmitting and overload are comprised of engaged mode of operation. The dynamic analysis of OWC mainly lies in the analysis of the characteristic of engaging, focusing on looking for the influence rule of the engaging time, the strut angle and the contact force between the sprags and races, which are influenced by the geometric parameters and the working condition of clutch.

Figure 3 gives the whole develop process of the dynamic equation of clutch.

3.1 Dynamic equation

Before the dynamic analysis, some assumptions are made as follows.

1) All sprags are wedged simultaneously, and there is no relative sliding between the sprags and the races.

2) The forces between the sprags and races can be regarded as continuous pressure, rather than concentrated force because of the large number of sprags.

3) Only the deformations of the inner race and the outer race are considered, and the sprags are considered as rigid body.

4) Compared to the Hertzian contact force, the centrifugal force of the sprags, the force between the
springs and the retainer, and the force between the springs and the spring can be neglected.

5) The inner ring and the outer ring are concentric definitely, without considering the manufacturing precision, the installation error of the spray, and the clearance between the spray and the raceways.

6) All springs share the load evenly.

Referring to the free body diagram of outer race shown in Fig. 4(a) and using D’Alembert’s principle, we can come to the resultant moment related to the center point O:

\[ -J_o\ddot{\phi}_o - nF_{fo}R_o + T = 0 \]  \( (9) \)

where \( J_o \) is the moment of inertia of the outer race, \( \phi_o \) is the angular displacement of the first contact point on the outer race, \( n \) is the total sprag number, \( F_{fo} \) is the tangential component of the contact force \( F_o \) and \( T \) is the torque load applied on the outer race.

According to Fig. 4(c) and D’Alembert’s principle, we can come to the resultant moment related to the center point O:

\[ -J_i\ddot{\phi}_i + nF_{fi}R_i - T = 0 \]  \( (10) \)

where \( J_i \) is the moment of inertia of the inner race, \( \phi_i \) is the angular displacement of the first contact point on the inner race, \( n \) is the total sprag number, \( F_{fi} \) is the tangential component of the contact force \( F_i \) and \( T \) is the torque load applied on the inner race.

According to Fig. 4(b) and D’Alembert’s principle, we can come to the resultant moment related to the first contact point on the inner cam:

\[ -J_i\ddot{\phi}_i - m_s a_s (R_i - R_o) - F_{ni}(R_i - R_o)\alpha_i + F_{fi}(R_i - R_o) = 0 \]  \( (11) \)

where \( J_i \) is the moment of inertia of a single sprag, \( \phi_i \) is the absolute angular displacement of a single sprag, \( m_s \) is the mass of a single sprag, \( a_s \) is the absolute acceleration of mass center of the sprag, \( \alpha_i \) is the inner strut angle, and \( F_{ni} \) is the normal component of the contact force. Because the inner strut angle \( \alpha_i \) is very small, the normal component of the inertial force \( m_s a_s^2 \) can be ignored compared to the normal contact force. Then, we can have the relation: \( F_{ni} = F_{no} \), \( a_s = a_s^i \).

XU and LOWEN [12] treated the motion of all
clutch components as an epicyclic train model in which the sprag is imagined as a planet on an imaginary arm rolls between a sun wheel (inner race) and a ring wheel (outer race). The local deformation between the sprags and the races provides the needed clearance for motion when the OWC is in engaging. Based on the geometry and kinematic relations of clutch components, we have [13]

\[ \phi_s = \frac{1}{R_i - R_o} (R_i \dot{\phi}_o - R_o \dot{\phi}_i) \]  

(12)

The concentric acceleration can be expressed approximately as

\[ a_s^i = \frac{a_o^i + a^i}{2} = \frac{R_i \dot{\phi}_o + R_o \dot{\phi}_i}{2} \]  

(13)

Substituting Eqs. (12) and (13) into Eq. (11) can work out the force \( F_i \). Because the inner strut angle \( \alpha \) is small, considering the balance of horizontal force of a single sprag, we can have the relation: \( F_i = F_{i0} - m_i a_s^i \).

Substituting the force \( F_i \) into Eq. (10), we have

\[ -J_i - \frac{n R_o^2}{(R_i - R_o)^2} \left( J_s - \frac{m_s}{4} (R_i - R_o)^2 \right) \dot{\phi}_o + \frac{n R_s R_o}{(R_i - R_o)^2} \left( J_s + \frac{m_s}{4} (R_i - R_o)^2 \right) \dot{\phi}_i + n \alpha_i R_o F_{m} = T \]  

(14)

Substituting the force \( F_{i0} \) into Eq. (9), we have:

\[ -J_o - \frac{n R_o^2}{(R_i - R_o)^2} \left( J_s - \frac{m_s}{4} (R_i - R_o)^2 \right) \dot{\phi}_o + \frac{n R_s R_o}{(R_i - R_o)^2} \left( J_s + \frac{m_s}{4} (R_i - R_o)^2 \right) \dot{\phi}_i - n \alpha_o R_o F_{m} = -T \]  

(15)

Using the following notations:

\[ A_1 = J_o + \frac{n R_o^2}{(R_i - R_o)^2} \left( J_s - \frac{m_s}{4} (R_i - R_o)^2 \right) \]
\[ A_2 = J_i + \frac{n R_s^2}{(R_i - R_o)^2} \left( J_s - \frac{m_s}{4} (R_i - R_o)^2 \right) \]
\[ A_3 = \frac{n R_s R_o}{(R_i - R_o)^2} \left( J_s + \frac{m_s}{4} (R_i - R_o)^2 \right) \]

The windup angle \( \theta \) is defined as the relative angular displacement between inner race and outer race. It is usually used in describing the dynamic engaging characteristics of OWC. Because the load torque \( T \) is on the inner race, \( \phi_s \) will be ahead of \( \phi_i \) all the times during operation, which can make the windup angle \( \theta = \phi_s - \phi_i \). Substitute \( A_1, A_2 \) and \( A_3 \) into Eqs. (14) and (15). Subtract \( A_2 \) Eq. (15) by \( A_1 \) Eq. (14) to get the first new equation, and subtract Eq. (15) by Eq. (14) to obtain the second new equation. Substituting the second new equation into the first new equation can obtain Eq. (16) as

\[(A_1 A_2 - A_3^2) \dot{\theta} + n \alpha_i F_{m} \left[ (A_1 - A_3) \cdot R_o + (A_2 - A_3) \cdot R_i \right] - T (A_1 + A_2 - 2 A_3) = 0 \]  

(16)

Equation (16) shows that there are two variables: one is the windup angle \( \theta \) and the other is the normal contact force \( F_{m} \). We can not work out the answer only by Eq. (16). However, as mentioned above, the deformation of the races provides the needed space for the movements between the sprags and the races, and because the torque \( T \) is applied on the inner race, the motion of the outer race will be ahead of the inner race, and there appears the windup angle \( \theta \). And we can have a conclusion that the greater the deformation emerges, the larger the windup angle generates. The amount of deformation is closely related with the contact force \( F_{m} \) between the sprag and the inner race. If we gain the relationship between the windup angle \( \theta \) and the contact force \( F_{m} \), and combine the Eq. (16), we can get the dynamic value of the windup angle \( \theta \).

3.2 Relationship between windup angle and deformation of races

The windup angle \( \theta \) can be considered to have two components (\( \theta = |\theta| + |\theta| \)). For the first part (\( |\theta| \)), it is assumed that the inner race is stationary and the motion is made by the deformation \( \delta_1 \) (see Fig. 5(a)), with the increase of the radius of the outer race. And for the second part \( |\theta| \), it is assumed that the outer race is stationary and the motion is only due to the deformation \( \delta_2 \) (see Fig. 5(b)), with the decrease of the radius of the inner race. In Fig. 5, the solid line stands for the initial position of the sprag, and the dashed one represents the final position. The small circles on the outlines indicate the respective initial and final contact points [12–13].

Using the calculation method of planetary gear transmission ratio, as XU and LOWEN [12] and CHANG [13] used, and assuming the inner race is stationary, we can have

\[ \theta_o = (1 + \frac{R_o}{R_i}) \varphi_{arm.o} \]  

(17)

where \( \varphi_{arm.o} \) shown in Fig. 5(a) is the rotation of the sprag around the center line of the races, with the imaginary of the stationary inner race and deformed outer race. As assumed above, the motions between the sprag and the races are pure rolling. With the deformation of \( \delta_1 \), the arc \( m \) on the inner race equals the distance that the sprag rotates around its center line, rolling along the inner race. The absolute value of the difference of rotation angle equals the angle that the sprag rotates around its own center line. Because the
Fig. 5 Rotation of sprag under contact deformation: (a) Deformation at outer interface with inner race stationary; (b) Deformation at inner interface with outer race stationary

angle of sprag revolution is very small, the radius of curvature of logarithmic spiral in two different contact states can be regarded to be equal. Making use of the equal arc length, we can have

$$\varphi_{\text{arm},o} = \frac{\rho \sqrt{1 + m^2}}{R_o} | \Delta \Omega_o | = \frac{\rho \sqrt{1 + m^2}}{R_o} | \overrightarrow{O} - \Omega |$$

(18)

where $\Omega$ is the rotation angle of the sprag at the initial contact state with the races, and $\overrightarrow{O}$ is the rotation angle of the sprag in the mode of engaged operation when the outer race is deformed. Equations (17) and (18) show that knowing the relationship between the deformation of the outer race $\delta_o$ and the difference of rotation angle of the sprag ($|\Delta \Omega_o|$), we can find out the relationship between the windup angle $\theta$ and the deformation of the outer race $\delta_o$. The undeformed radius of inner surface of outer race $R_i$ with the deformed radius of inner surface of outer race $(R_i + \delta_o)$ in Eqs. (4)–(8) is replaced to get the deformed value of $\overrightarrow{FA}, \overrightarrow{FB}, \overrightarrow{FC}, \overrightarrow{FD}$, and $\overrightarrow{OE}$. As the difference of the relative roll angle ($|\Delta \Omega_o|$) is only affected little by the deformation of outer race $\delta_o$, it can be simplified by using Maclaurin series:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

(19)

Ignoring the two-order and two-order above, Eq. (19) becomes

$$f(x) \approx f(0) + f'(0)x$$

(20)

By using the following notations and applying Eq. (20), we have

$$x_o = \frac{mR_o}{(R_i + \delta_o - r_o) \sqrt{1 + m^2}}$$

$$x = x_o(\delta_o = 0) = \frac{mR_o}{(R_i - r_o) \sqrt{1 + m^2}}$$

$$\Delta \Omega_o = f(\delta_o) \approx f(0) + \frac{\text{d} \Delta \Omega_o}{\text{d} \delta_o} |_{\delta_o = 0} \cdot \delta_o$$

$$\frac{\text{d} \Delta \Omega_o}{\text{d} \delta_o} |_{\delta_o = 0} \cdot \delta_o = \frac{\text{d} \Delta \Omega_o}{\text{d} \beta_o} \frac{\text{d} \beta_o}{\text{d} \delta_o} |_{\delta_o = 0} \cdot \delta_o - \frac{\text{d} \Delta \Omega_o}{\text{d} \phi_o} \frac{\text{d} \phi_o}{\text{d} \delta_o} |_{\delta_o = 0} \cdot \delta_o$$

$$= \frac{\text{d} \beta_o}{\text{d} \delta_o} |_{\delta_o = 0} \cdot \delta_o - \frac{\text{d} \phi_o}{\text{d} \delta_o} |_{\delta_o = 0} \cdot \delta_o$$

$$= 1 + m \sqrt{1 - x^2 - x^2}$$

(21)

$$\theta = \frac{1}{R_i} \cdot \rho \sqrt{1 + m^2} \cdot B_o \cdot \delta_o$$

(22)

Similarly, we can derive the corresponding
equations for inner race, when it is assumed that the outer race is fixed and the inner race has deformed as shown in Fig. 5(b).

\[
\Delta \Omega = f(\delta_i) \approx f(0) + \frac{d\Delta \Omega}{d\delta_i}|_{\delta_i=0} \cdot \delta_i
\]

\[
= -\frac{1}{(R_i-r_o)\sqrt{1+m^2}} \left( m\sqrt{1-x^2} + m^2 \right) \cdot \delta_i
\]

\[
= B_i \cdot \delta_i
\]

Then, we will have

\[
| \theta_i | = \delta_i = C_i \cdot \delta_i
\]

Equation (25) shows the relationship between the windup angle \( \theta \) and the deformation of races \( \delta \). It is needed to derive the relationship between the normal contact forces between the sprags and races \( F_n \) and the deformation of races \( \delta \) to find the solution of Eq. (16).

### 3.3 Relationship between normal contact force and deformation

When the OWC is just at the beginning of the mode of engaging operation, the outer race with initial velocity collides with the static sprag, and there exists transient impact force at the interface between the sprags and the races. The sprag will collide repeatedly to the races before it is fully engaged and there will be an indication of vibration. During this stage, the torque applied on the inner race increases from zero to the maximum value, and the deformation of races increases from zero to the maximum value. The contact force achieves steady value at the moment that the sprag is fully engaged between the races.

The contact force between the sprags and races during the mode of engaging operation is similar with the mode of engaging operation, the outer race with initial velocity collides with the static sprag, and there exists transient impact force at the interface between the sprags and the races. The sprag will collide repeatedly to the races at the moment that the sprag is fully engaged between the sprags and the races. With the small value of the inner strut angle and the outer strut angle, we can consider that the contact force \( F_{no} \) equals the normal contact force between the sprag and the races. \( K_n \) is the contact stiffness between the sprag and the races, and \( F_{ni} \) is the contact stiffness between the sprag and the inner race. The normal contact force consists two parts: the contact stiffness between the sprag and the outer race, and the contact stiffness between the sprag and the inner race.

The calculation of correlation coefficient is as follows:

\[
K = \frac{4}{3(\eta_1 + \eta_2)} \left( \frac{R_1 R_2}{R_1 \pm R_2} \right)^{\frac{3}{2}}, \quad \eta_1 = 1 - \nu_1^2, \quad \eta_2 = 1 - \nu_2^2
\]

where \( \eta_1 \) and \( \eta_2 \) are the curvature radii of two objects, \( \nu_1 \) and \( \nu_2 \) are Poisson ratios of two objects, \( E_1 \) and \( E_2 \) are elastic modulus, and \( K \) is the contact stiffness.

There are two possible forms of contact between a race and sprag: 1) when the convex contact with the convex is just between the inner race and sprag, we can replace the symbol(±) with the symbol(+) in Eqs. (27) and 2) when the concave contact with the convex is just between the outer race and sprag, we can replace the symbol(±) with the symbol(−). And \( \alpha \) is empirical coefficient, which can be assigned to 0.08–0.32 m/s [23], and we have Eq. (28) as

\[
F_{no} = K_n \delta_2^3 + \xi_n \delta_0^3 \delta_1
\]

where \( F_{no} \) and \( F_{ni} \) are the normal contact force between the sprag and the races. With the small value of the inner strut angle and the outer strut angle, we can consider that the contact force \( F_{no} \) equals the contact force \( F_{ni} \). \( K_i \) is the contact stiffness between the sprag and the inner race. The normal contact force consists two parts: the contact stiffness between the sprag and the outer race, and the contact stiffness between the sprag and the inner race.

It can be derived by Eq. (28) and we have

\[
\delta_i = \left( \frac{F_{nih}}{K_i} \right)^{\frac{2}{3}}
\]

Substituting Eq. (29) into Eq. (25), we can develop the relationship between the normal spring contact force and the inner strut angle as

\[
F_{nih} = \frac{G \theta^{3/2}}{\left( \frac{C_n}{K_o^{2/3}} + \frac{C_i}{K_i^{2/3}} \right)^{1/2}}
\]

Similarly, we can develop the relationship between the damping force and the inner strut angle as
Substituting Eqs. (30) and (31) into Eq. (16), we have

\[
(A_i A_2 - A_3^2) \dot{\theta} + n \alpha_i C_{nl} [(A_2 - A_3) R_i + (A_i - A_3) R_o] \theta^p \dot{\theta} + n \alpha_i G[(A_2 - A_3) R_i + (A_i - A_3) R_o] \theta^{3/2} - T(A_i + A_2 - 2A_3) = 0
\]  

(32)

\[\text{Substituting Eqs. (30) and (31) into Eq. (16), we have}\]

\[F_{nl} = \frac{(p+1)\theta^p \dot{\theta}}{(C_o \left(\frac{p+1}{\xi_o} + C_i \left(\frac{p+1}{\xi_i}\right)^{p+1}\right)^{p+1}} = C_{nl} \theta^p \dot{\theta} \quad (31)\]

4 Results and discussion

Substitute the data in Table 1 into the relevant equation mentioned above and analyze the dynamic change rule of the windup angle, the engaging time, the transient impact force and steady contact force with different inner strut angles and different initial collision velocities of outer race.

4.1 Rule of windup angle variation with different initial velocity of outer race

Figure 6 shows the influence rule of the logarithmic spiral type sprag clutch, of which the inner strut angle is 3.2°, and it is assumed that the outer race has different initial velocities when the clutch is in the mode of engaging operation. We can obtain the dynamic change rule of the maximum transient windup angle, the steady windup angle and steady engaging time with different initial velocities of outer race, as shown in Fig. 7. It can

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of inner surface of outer race, (R_i/mm)</td>
<td>45.5995</td>
</tr>
<tr>
<td>Radius of outer surface of outer race/mm</td>
<td>58</td>
</tr>
<tr>
<td>Radius of outer surface of inner race, (R_o/mm)</td>
<td>36.106</td>
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<tr>
<td>Radius of inner surface of inner race/mm</td>
<td>25</td>
</tr>
<tr>
<td>Length of inner and outer race, (L/mm)</td>
<td>22</td>
</tr>
<tr>
<td>Radius of outer cam of sprag, (r_o/mm)</td>
<td>4.9233</td>
</tr>
<tr>
<td>Number of sprags</td>
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<tr>
<td>Mass of a sprag, (m_s/g)</td>
<td>9.427</td>
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<tr>
<td>Inner strut angle, (\alpha_i/rad)</td>
<td>0.0645</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque/(N·m)</td>
<td>1326.25</td>
</tr>
<tr>
<td>Sprag material</td>
<td>GCr15</td>
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<tr>
<td>Poisson ratio, (\nu_1)</td>
<td>0.29</td>
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<tr>
<td>Elastic modulus, (E_1/GPa)</td>
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</tr>
<tr>
<td>Length of sprag, (L_s/mm)</td>
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</tr>
<tr>
<td>Inner and outer ring material</td>
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<tr>
<td>Poisson ratio, (\nu_2)</td>
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<tr>
<td>Elastic modulus, (E_2/GPa)</td>
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<tr>
<td>Initial rotation angle, (\Omega/rad)</td>
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</tbody>
</table>

**Fig. 6** Change rule of windup angle with different velocities of outer race: (a) \(\alpha_i=3.2^\circ, p=1.5, v_o=31.4\) rad/s; (b) \(\alpha_i=3.2^\circ, p=1.5, v_o=94.2\) rad/s; (c) \(\alpha_i=3.2^\circ, p=1.5, v_o=188.4\) rad/s; (d) \(\alpha_i=3.2^\circ, p=1.5, v_o=376.8\) rad/s
be found that the maximum transient windup angle and the steady engaging time increase linearly nearly with initial velocity of outer race, while the steady windup angle keeps a constant value nearly.

### 4.2 Frequency characteristic analysis of clutch under different initial velocity conditions

Figure 8 shows the frequency analysis of step dynamic response of the clutch in the mode of engaging operation, of which the inner strut angle is 3.2° and the velocity of the outer race is 31.4 rad/s. It can be found that near the frequency of 585 Hz, which is the natural frequency in fact, the system has the maximum energy output. It is also known that the natural cycle is 0.0017 s. Figure 9 illustrates that the natural frequency and cycle remain constant when the clutch is in the mode of engaging operation with different initial velocities of outer race.

### 4.3 Rule of contact impact force variation with different initial velocities of outer race

When the clutch is in the mode of engaging operation, there exists collisions between static sprags and the outer race with initial velocity. Normal contact impact force appears between the sprags and races before the sprags are fully wedged, and the force remains constant after the sprags are fully wedged. Figure 10 shows the change rule of the normal contact force between a sprag and inner race with different velocities of outer race and the inner strut angle of 3.2°. It can be found in Fig. 11 that the steady contact force remains constant with nothing to do with the initial velocity of outer race, while the maximum transient contact impact force increases approximately linearly with the initial velocity of the outer race.

### 4.4 Change rule of windup angle with different inner strut angles

In order to explore the engagement characteristics of clutch influenced by the inner strut angle, the dynamic change rules of the windup angle and normal contact force between sprag and races are analyzed under the condition of the same velocity of outer race. It can be found in Fig. 12 that the steady windup angle, the maximum transient windup angle and the steady engaging time decrease with the inner strut angle of clutch.

### 4.5 Frequency characteristics analysis of clutch in condition of different inner strut angles

Figure 13 shows the change rule of the natural frequency and cycle of the clutch under the condition of the same initial velocity of outer race (31.4 rad/s) and different inner strut angles. It can be found that the natural frequency increases with the inner strut angle and the natural cycle decreases with it.

### 4.6 Change rule of normal contact force in condition of different inner strut angles

Figure 14 shows the change rule of the maximum
Fig. 10 Change rules of normal force between sprag and races with different velocities of outer race: (a) $\alpha=3.2^\circ$, $p=1.5$, $v_o=31.4$ rad/s; (b) $\alpha=3.2^\circ$, $p=1.5$, $v_o=94.2$ rad/s; (c) $\alpha=3.2^\circ$, $p=1.5$, $v_o=282.6$ rad/s; (d) $\alpha=3.2^\circ$, $p=1.5$, $v_o=376.8$ rad/s

Fig. 11 Change rule of contact force with velocity of outer race

Fig. 12 Change rule of windup angle and engaging time with inner strut angle

Fig. 13 Rule of natural frequency and cycle with inner strut angle

Fig. 14 Rule of contact force with inner strut angle
transient impact contact force and steady contact force under condition of different inner strut angles and the same initial velocity of outer race of 31.4 rad/s. It can be found that the maximum transient impact contact force and the steady contact force decrease with the inner strut angle.

5 Conclusions

1) In order to guarantee the normal operation of the OWC, the sprag must be satisfied with the condition of self-locking, and the inner strut angle should not exceed 4.574°.

2) The parameters of the steady windup angle, the steady contact force, the natural frequency and the natural cycle of the system are not affected by the initial velocity of outer race. While the maximum windup angle, the maximum transient impact force and the steady wedging time increase nearly linearly with the initial collision velocity of the out race.

3) The inner strut angle has great influence on the dynamic wedging performance. The parameters of the steady windup angle, the maximum transient impact angle, the steady engaging time, the steady contact force, the maximum transient impact force and the natural cycle of the system decrease linearly nearly with the inner strut angle, while the natural frequency of the system increases linearly nearly with the inner strut angle.

References


(Edited by FANG Jing-hua)