Interaction due to various sources in saturated porous media with incompressible fluid

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Abstract: The disturbance due to mechanical and thermal sources in saturated porous media with incompressible fluid for two-dimensional axi-symmetric problem is investigated. The Laplace and Hankel transforms techniques are used to investigate the problem. The concentrated source and source over circular region have been taken to show the utility of the approach. The transformed components of displacement, stress and pore pressure are obtained. Numerical inversion techniques are used to obtain the resulting quantities in the physical domain and the effect of porosity is shown on the resulting quantities. All the field quantities are found to be sensitive towards the porosity parameters. It is observed that porosity parameters have both increasing and decreasing effect on the numerical values of the physical quantities. Also the values of the physical quantities are affected by the different boundaries. A special case of interest is also deduced.

Key words: axi-symmetry; incompressible porous medium; pore pressure; Laplace transform; Hankel transform; concentrated source and source over circular region

1 Introduction

The dynamic response due to various sources in a saturated porous media with incompressible fluid are of great interest in geophysics, acoustic, soil and rock mechanics and many earthquake engineering problems.

BIOT [1] derived the basic equations of poroelasticity on the basis of energy principles. PREVOST [2] rederived these equations by use of mixture theory. ZENKIEWICZ et al [3], ZENKIEWICZ and SHIOMI [4] derived the basic equations of poroelasticity by the use of principle of continuum mechanics. GATMIRI and KAMALIAN [5] adopted the later approach because it is more flexible and is based on a set of parameters with a clear physical interpretation to discuss different type of problems. GATMIRI and NGUYEN [6] investigated two-dimensional problem for saturated porous media with incompressible fluid.


OLIVEIRA et al [17] discussed boundary element formulation of axisymmetric problems for an elastic half space. The influence of the finite initial strains on

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the axisymmetric wave dispersion in a circular cylinder embedded in a compressible elastic medium was discussed by AKBAROV and GULIEV [18]. GORDELIY and DETOURNA [19] investigated the displacement discontinuity method for modelling axisymmetric cracks in an elastic half-space. JABBARI and DEHBANI [20–21] studied axisymmetric and spherical symmetric problems in porothermoelastic solids. ABBAS [22] studied the natural frequencies of a poroelastic hollow cylinder. For our contribution, several problems have been solved by finite element method and analytical method [23–31]. LIU and CHEN [32] studied the problem of a micromechanical analysis of the fracture properties of saturated porous media.

In the present work, the disturbance is due to concentrated source and source over circular region in the time domain and frequency domain in saturated porous media with incompressible fluid. The transformed components of displacement, stress and pore pressure are obtained and depicted graphically to show the effect of porosity on the resulting quantities.

2 Governing equations

Following GATMIRI and NGUYEN [6], the field equations are as follows.

Equation of motion:
\[ \sigma_{ij,j} + f_i = \rho u_i + \rho_i \dot{W}_i \] (1)

Generalized Darcy’s law:
\[ p_j = \frac{1}{k} \dot{W}_j - \rho_i \dot{u}_j - m \dot{W}_j \] (2)

Constitutive relation:
\[ \sigma_{ij} = \lambda u_{k,k} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) - \alpha \rho \] (3)

Flow conservation for the fluid phase:
\[ -\dot{W}_{ij,j} + \gamma = \alpha \dot{u}_{k,k} + \frac{\dot{p}}{M} \] (4)

where \( u_i \) is the displacement of the solid skeleton; \( \rho \) denotes the fluid pressure; \( \dot{W}_i \) represents the average displacement of the fluid relative to the solid. The elastic constants \( \lambda \) and \( \mu \) are Lame’s constants. \( \rho_f \) is the fluid density; \( \rho_i \) is the solid density; \( \rho = (1-n)\rho_s + n\rho_f \) is the density of solid-fluid mixture and \( m = \rho_f/n \) is the mass parameter where \( n \) is the porosity; \( k \) is the permeability coefficient. \( \alpha \) and \( M \) are material parameters which describe the relative compressibility of the constituents. \( f_i \) and \( \gamma \) denote the body force and the rate of fluid injection into the media can be reduced to
\[ \mu u_{i,jj} + (\lambda + \mu) u_{k,kj} - \rho_i \dot{u}_j - \alpha \dot{p} = 0 \] (5)
\[ \rho M \frac{\partial^2 \dot{p}}{\partial t^2} - \alpha \dot{u}_{k,k} = 0 \] (6)

where
\[ \rho_i = \rho - \rho_i^2 \tau \frac{\partial}{\partial t}, \alpha = \alpha - \rho_i \tau \frac{\partial}{\partial t}, \tau = \left[ 1 + m \frac{\partial}{\partial t} \right]^{-1}. \]

3 Formulation of problem

We consider a saturated porous media with incompressible fluid whose boundaries are parallel to the plane \( z=0 \) in the cylindrical polar coordinate system (\( r, \theta, z \)). We consider a two dimensional axisymmetric problem with symmetry about \( z \)-axis, so that all the quantities are maintained independent of \( \theta \) and \( \partial / \partial \theta = 0 \). The complete geometry of the problem is shown in Figs. 1(a) and (b). We assume the components of displacement vector as
\[ u = (u_r, 0, u_z) \] (7)
where \( \omega \) is the constant having the dimensions of frequency.

The displacement components \( u_z \) and \( u_r \) are related to the potential functions \( \Phi \) and \( \Psi \) as

\[
\begin{align*}
    u_z &= \frac{\partial \Phi}{\partial r} - \frac{\partial \Psi}{\partial z} \\
    u_r &= \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial r}
\end{align*}
\]  

(9)

We define the Laplace and Hankel transforms as follows:

\[
\tilde{f}(s) = \int_0^\infty f(t) e^{-st} dt
\]  

(10)

\[
\tilde{f}(\xi, z, s) = \int_0^\infty f(\tau, z, s) J_n(\tau \xi) d\tau
\]  

(11)

where \( J_n(\cdot) \) is the Bessel function of the first kind of index \( n \).

Making use of the dimensionless quantities defined by Eq. (8) on Eqs. (5) and (6) and with the aid of Eqs. (7) and (9) (after suppressing the prime) and applying the Laplace and Hankel transforms defined by Eqs. (10) and (11) on the resulting quantities after simplification, we obtain

\[
\left( \frac{d^4}{dz^4} - A_1 \frac{d^2}{dz^2} + B_1 \right) \tilde{\Phi} = 0
\]  

(12)

\[
-\xi^2 \tilde{\Psi} + \frac{d^2 \tilde{\Psi}}{dz^2} - A_2 \tilde{\Psi} = 0
\]  

(13)

where

\[
A_1 = B_1 + B_1 + A_2 A_3 + 2 \xi^2,
\]

\[
B_1 = \frac{\xi^2}{s^2} + (B_1 + B_1 + A_2 A_3) \xi^2 + B_1 B_2,
\]

\[
A_1 = \frac{a_2}{1 + a_1}, \quad A_2 = \frac{a_3 s^2}{a_1}, \quad A_3 = \frac{s}{b_1},
\]

\[
B_1 = \frac{a_3 s^2}{1 + a_1}, \quad B_2 = \frac{b_3 s}{b_1},
\]

\[
a_1 = \frac{\mu}{\lambda + \mu}, \quad a_2 = \frac{\alpha', \lambda}{\lambda + \mu}, \quad a_3 = \frac{c^2_0 P_1}{\lambda + \mu},
\]

\[
b_1 = \frac{\mu \alpha'}{\lambda + \mu}, \quad b_2 = \frac{\lambda}{\alpha' M'}
\]

Using the solution of Eqs. (13) and (15) and satisfying the radial conditions that \( \tilde{\Phi}, \tilde{\Psi}, \) and \( \tilde{p} \) \( \to 0 \) as \( z \to \infty \), we obtain the values of \( \tilde{\Phi}, \tilde{\Psi}, \) and \( \tilde{p} \) as follows:

\[
\tilde{\Phi} = C_1 e^{-m_1 z} + C_2 e^{-m_2 z}
\]  

(14)

\[
\tilde{\Psi} = E_1 e^{-m_1 z}
\]  

(15)

\[
\tilde{p} = r_1 C_1 e^{-m_1 z} + r_2 C_2 e^{-m_2 z}
\]  

(16)

where \( m_1 \) and \( m_2 \) are given by

\[
m_n^2 = A_1 + \sqrt{A_1^2 - 4B_1}, \quad \text{when } n=1, 2, \text{ are the roots of Eq. (15)}; \quad m_1 = \sqrt{A_4}, \quad \text{where } A_4 = A_2 + \xi^2, \quad \text{and the coupling constants are given by}
\]

\[
r_i = \frac{m_i^2 - B_1 - \xi^2}{A_1}, \quad i=1, 2.
\]

4 Boundary conditions and solution of problem

The boundary conditions at \( z=0 \) are

\[
\begin{align*}
    \sigma_{zz} &= -P_1 F(r) \delta(t) \\
    \sigma_{zr} &= -P_3 F(r) \delta(t) \\
    p &= P_2 F(r) \delta(t)
\end{align*}
\]  

(17)

where \( P_1 \) and \( P_3 \) are the magnitudes of the forces; \( P_2 \) is the constant pressure applied on the boundary; \( F(r) \) is the known functions defined later in the manuscript.

Applying Laplace and Hankel transforms, we have

\[
\begin{align*}
    \tilde{\sigma}_{zz} &= -P_1 \tilde{F}(\xi) \\
    \tilde{\sigma}_{zr} &= -P_3 \tilde{F}(\xi) \\
    \tilde{p} &= P_2 \tilde{F}(\xi)
\end{align*}
\]  

(18)

The stress components are obtained with the aid of Eqs. (3), (7), (10), (11), (16) and (17) as

\[
\tilde{\sigma}_{zz} = \tilde{F}(\xi)(-\xi a_1 e^{-m_1 z} - \xi a_{22} e^{-m_2 z} + m_1 a_{33} e^{-m_1 z})
\]  

(19)

\[
\tilde{\sigma}_{zr} = \tilde{F}(\xi)(a_1 a_{11} e^{-m_1 z} - a_{32} e^{-m_2 z} + m_{13} a_{33} e^{-m_1 z})
\]  

(20)

\[
\tilde{\sigma}_{zz} = \tilde{F}(\xi)(d_4 a_{11} e^{-m_1 z} + d_{12} a_{22} e^{-m_2 z} + d_{13} a_{33} e^{-m_1 z})
\]  

(21)

\[
\tilde{\sigma}_{zr} = \tilde{F}(\xi)(d_4 a_{11} e^{-m_1 z} + d_{12} a_{22} e^{-m_2 z} + d_{13} a_{33} e^{-m_1 z})
\]  

(22)

\[
\tilde{\sigma}_{zz} = \tilde{F}(\xi)(r_1 a_1 e^{-m_1 z} + r_2 a_{22} e^{-m_2 z})
\]  

(23)

where

\[
d_1 = -\xi^2 - a_1 R_2 m_1^2, \quad d_2 = -\xi^2 - a_2 R_2 m_2^2, \quad d_3 = 2\xi m_3, \quad d_4 = 2\xi m_1 R_1, \quad d_5 = 2\xi m_2 R_1, \quad d_6 = -(m_1^2 + \xi^2) R_1,
\]

\[
a_{11} = -P_1 r_2 d_6 - P_3 d_3 d_6 + P_2 d_3 d_6 + P_3 r_2 d_3,
\]

\[
a_{22} = P_2 d_3 d_6 - P_1 r_1 d_6 - P_3 r_1 d_6 - P_2 d_3 d_6 + P_3 r_2 d_3,
\]

\[
a_{33} = -P_2 r_1 d_4 - P_3 r_4 d_4 + P_2 d_4 d_3 + P_3 r_2 d_3 - P_2 r_1 d_4 + P_3 r_2 d_3,
\]

\[
a_{44} = -P_2 r_1 d_6, \quad a_{55} = -P_3 r_1 d_6, \quad a_{66} = -P_3 r_1 d_6 + P_3 r_2 d_4,
\]

\[
A = \frac{1}{\lambda + \mu}, \quad A_1 = \frac{1}{a_1}, \quad A_2 = \frac{1}{a_1}, \quad A_3 = \frac{1}{a_1}, \quad A_4 = \frac{1}{a_1}, \quad A_5 = \frac{1}{a_1}, \quad A_6 = \frac{1}{a_1}.
\]
5 Case study

5.1 Case 1: Concentrated source

The solution due to concentrated source is obtained by substituting

\[ F(r) = \frac{1}{2\pi r} \delta(r) \]  

Applying Laplace and Hankel transforms on Eq. (24), we obtain

\[ \tilde{F}(\zeta) = \frac{1}{2\pi} \]

5.2 Case 2: Source over circular region

The solution due to source over the circular region of non-dimensional radius \( a \) is obtained by setting \( F(r) = \frac{1}{2\pi r} \delta(r) \).

Applying Laplace and Hankel transforms on these quantities, we obtain

\[ \tilde{F}(\zeta) = \frac{1}{2\pi \xi} J_1(\alpha \zeta) \]

5.3 Frequency domain

In this case we assume the time harmonic behaviour as

\[ (u_1^i, u_2^i, p)(r, z, t) = (u_1^i, u_2^i, p)(r, z, e^{\text{i}\omega t}), i = F, S \]

and the boundary conditions Eq. (19) takes the form:

\[ \sigma_{zz} = -P_F(r)e^{\text{i}\omega t}, \sigma_{zz} = -P_F(r)e^{\text{i}\omega t}, \sigma = P_F F(r)e^{\text{i}\omega t} \]

The expressions for displacement, stress and pore pressure in frequency domain can be obtained by replacing \( s \) by \( \text{i}\omega \) in the expressions of time domain Eqs. (2)–(23).

5.4 Special case

In the absence of porous incompressible fluid, the boundary conditions can be reduced to \( \tilde{\sigma}_{zz} = -P_F \tilde{\zeta} \).

\[ \tilde{\sigma}_{zz} = -P_F \tilde{\zeta} (\xi, \omega), \] and the corresponding expressions for stress components in elastic half space, we obtain

\[ \tilde{\sigma}_{zz} = \tilde{F}(\zeta)[d_3 b_{44}e^{-\text{i}\omega z} + d_8 b_{55}e^{-\text{i}\omega z}] \]  

\[ \tilde{\sigma}_{zz} = [d_3 b_{44}e^{-\text{i}\omega z} + d_8 b_{55}e^{-\text{i}\omega z}] \]

where

\[ d_7 = -\xi^2 + R_2 m_z^2, \quad d_8 = (1 - R_2) \xi m_z, \quad d_9 = 2 \xi m_R, \]

\[ b_{44} = \frac{(-P_F d_6 + P_F d_2)}{\Delta_0}, \quad b_{55} = \frac{(-P_F d_7 + P_F d_9)}{\Delta_0}, \]

\[ \Delta_0 = d_6 d_7 - d_8 d_9. \]

Taking \( P_F = 0 \) and \( P_I = 0 \) in Eqs. (25)–(26), we obtain respectively the stress components for the normal and tangential forces.

5.5 Numerical results and discussion

For numerical computation, the following values of the various physical parameters are taken from GATMIRI and NGUYEN [6]:

\[ \lambda = 12.5 \text{ MPa}, \mu = 8.33 \text{ MPa}, K_F = 10^7 \text{ MPa}, \]

\[ K_F = 10^7 \text{ MPa}, \rho_s = 2600 \text{ kg/m}^3, \rho_F = 1000 \text{ kg/m}^3, \]

\[ k = 0.001 \text{ m/s}, \alpha = 1, n = 0.3, \omega = 1. \]

The values of normal stress \( \sigma_{zz} \), tangential stress \( \tau_{zz} \) and pore pressure \( p \) for fluid saturated incompressible porous medium (FSPM) and empty porous medium (EPM) are shown due to concentrated source and source applied over the circular region. The computation is carried out for two values of dimensionless time \( t = 0.50 \) and \( t = 0.75 \) at \( z = 1 \) in the range \( 0 \leq r < 10 \).

The solid lines either without central symbols or with central symbols represent the variations for \( t = 0.1 \), whereas the dashed lines with or without central symbols represent the variations for \( t = 0.50 \). Curves without central symbols correspond to the case of FSPM whereas those with central symbols correspond to the case of EPM.

5.6 Time domain

Figure 2 shows the variation of normal stress component \( \sigma_{zz} \) w.r.t distance \( r \) for both FSPM and EPM due to concentrated normal force. The value of \( \sigma_{zz} \) first increases in the range \( 0 \leq r \leq 7 \) and then starts decreasing for FSPM as \( r \) increases and in the case of EPM, its value oscillates as \( r \) increases for both values of time.

Figure 3 shows the variation of normal stress component \( \sigma_{zz} \) w.r.t distance \( r \) for both FSPM and EPM due to concentrated tangential force. The value of \( \sigma_{zz} \) oscillates for FSPM as \( r \) increases for both values of time whereas in the case of EPM, the value of \( \sigma_{zz} \) starts with sharp decrease and then oscillates for both values of time as \( r \) increases.
Figure 2 shows the variation of normal stress $\sigma_{zz}$ with distance $r$ due to concentrated normal force. The value of $\sigma_{zz}$ is more in the range $3 \leq r \leq 6$ and less in the range $6 \leq r \leq 9$ for FSPM as $r$ increases for time $t=0.50$ whereas for the time $t=0.75$ its value converges near the boundary surface.

Figure 3 shows the variation of normal stress $\sigma_{zz}$ with distance $r$ due to concentrated tangential force. The value of $\sigma_{zz}$ decreases sharply for FSPM and then oscillates as $r$ increases for both values of time.

Figure 4 shows the variation of normal stress $\sigma_{zz}$ with distance $r$ due to concentrated pressure source. The value of $\sigma_{zz}$ decreases in the range $0 \leq r \leq 3.5$ and then oscillates for FSPM as $r$ increases for time $t=0.50$ whereas for the time $t=0.75$ its value converges near the boundary surface.

Figure 5 shows the variation of pore pressure $p$ with distance $r$ due to concentrated normal force. The value of $p$ decreases in the range $0 \leq r \leq 3.5$ and then oscillates for FSPM as $r$ increases for time $t=0.50$ whereas for the time $t=0.75$ the value of $p$ decreases and then oscillates as $r$ increases.

Figure 6 shows the variation of pore pressure $p$ with distance $r$ due to concentrated tangential force. The value of $p$ decreases sharply for FSPM and then oscillates as $r$ increases for both values of time.

Figure 7 shows the variation of pore pressure $p$ with distance $r$ due to concentrated pressure source. The value of $p$ converges near the boundary surface for FSPM as $r$ increases for time $t=0.50$ whereas for the time $t=0.75$ the value of $p$ increases sharply and then...
oscillates as \( r \) increases.

Figure 8 shows the variation of tangential stress \( \sigma_{zr} \) w.r.t distance \( r \) for both FSPM and EPM due to concentrated normal force. The value of \( \sigma_{zr} \) first decreases in the range \( 0 \leq r \leq 3 \) and then oscillates for FSPM as \( r \) increases for both value of time whereas the value of \( \sigma_{zr} \) decreases sharply and then oscillates for EPM as \( r \) increases for both value of time.

![Fig. 7](image7.png)
Fig. 7 Variation of pore pressure \( p \) with distance \( r \) due to concentrated pressure source

![Fig. 8](image8.png)
Fig. 8 Variation of tangential stress \( \sigma_{zr} \) with distance \( r \) due to concentrated normal force

Figure 9 shows the variation of tangential stress \( \sigma_{zr} \) w.r.t distance \( r \) for both FSPM and EPM due to concentrated tangential force. The value of \( \sigma_{zr} \) first increases monotonically and then converges near the boundary surface for FSPM and for EPM its value almost closes zero as \( r \) increases for both values of time.

![Fig. 9](image9.png)
Fig. 9 Variation of tangential stress \( \sigma_{zr} \) with distance \( r \) due to concentrated tangential force

Figure 10 shows the variation of normal stress \( \sigma_{zz} \) w.r.t distance \( r \) for both FSPM and EPM due to normal force over circular region. The value of \( \sigma_{zz} \) first increases in the range \( 0 \leq r \leq 7 \) and then starts decreasing for FSPM as \( r \) increases and in the case of EPM, its value oscillates as \( r \) increases for both values of time.

![Fig. 10](image10.png)
Fig. 10 Variation of tangential stress \( \sigma_{zr} \) with distance \( r \) due to concentrated pressure source

Figure 11 shows the variation of normal stress \( \sigma_{zz} \) w.r.t distance \( r \) for both FSPM and EPM due to normal force over circular region. The value of \( \sigma_{zz} \) first increases in the range \( 0 \leq r \leq 7 \) and then starts decreasing for FSPM as \( r \) increases and in the case of EPM, its value oscillates as \( r \) increases for both values of time.

![Fig. 11](image11.png)
Fig. 11 Variation of normal stress \( \sigma_{zz} \) with distance \( r \) due to concentrated normal force

Figure 12 shows the variation of normal stress \( \sigma_{zz} \) w.r.t distance \( r \) for both FSPM and EPM due to tangential force over circular region. The value of \( \sigma_{zz} \) starts oscillating for FSPM as \( r \) increases for both values of time and the value of \( \sigma_{zz} \) for EPM first decreases and then starts oscillating as \( r \) increases for both values of time.

![Fig. 12](image12.png)
Fig. 12 Variation of normal stress \( \sigma_{zz} \) with distance \( r \) due to concentrated tangential force

Figure 13 shows the variation of normal stress \( \sigma_{zz} \) w.r.t distance \( r \) for FSPM due to pressure source over circular region. The value of \( \sigma_{zz} \) first increases in the
range $0 \leq r \leq 6$ then decreases for FSPM as $r$ increases for both values of time.

Figure 14 shows the variation of pore pressure $p$ w.r.t distance $r$ for FSPM due to normal force over circular region. The value of $p$ decreases sharply in the range $0 \leq r \leq 3.5$ and then oscillates as $r$ increases for both values of time.

Figure 15 shows the variation of pore pressure $p$ w.r.t distance $r$ for FSPM due to tangential force over circular region. The value of $p$ remains almost close to zero for FSPM as $r$ increases for time $t=0.5$ whereas for the time $t=0.75$ the value of $p$ decreases sharply and then oscillates as $r$ increases.

Figure 16 shows the variation of pore pressure $p$ w.r.t distance $r$ for FSPM due to pressure source over circular region. The value of $p$ starts with initial increase and then oscillates for FSPM as $r$ increases for time $t=0.5$ whereas for the time $t=0.75$ the value of $p$ increases sharply in the range $0 \leq r \leq 5$ and then starts decreasing as $r$ increases.
Figure 17 shows the variation of tangential stress $\sigma_r$ w.r.t distance $r$ for both FSPM and EPM due to normal force over circular region. The value of $\sigma_r$ first decreases and then oscillates for FSPM as $r$ increases for both value of time whereas the value of $\sigma_r$ decreases gradually and then oscillates for EPM as $r$ increases for both value of time.

Figure 18 shows the variation of tangential stress $\sigma_r$ w.r.t distance $r$ for both FSPM and EPM due to tangential force over circular region. The value of $\sigma_r$ first increases monotonically and then remains almost close to zero for FSPM for both values of time and first increases monotonically and then converges near the boundary surface for the time $t=0.75$ for EPM.

Figure 19 shows the variation of tangential stress $\sigma_r$ w.r.t distance $r$ for both FSPM and EPM due to pressure source over circular region. The value of $\sigma_r$ decreases sharply and then oscillates as $r$ increases for FSPM for both value of time.

5.7 Frequency domain

Figure 20 shows the variation of normal stress component $\sigma_z$ w.r.t distance $r$ for both FSPM and EPM due to concentrated normal force. The value of $\sigma_z$ first increases monotonically in the range $0 \leq r < 2$ and then start oscillating for FSPM as $r$ increases for both values of time where its value oscillates as $r$ increases for $t=0.50$ and converges near the boundary surface for $t=0.75$ for EPM.

Figure 21 shows the variation of normal stress component $\sigma_z$ w.r.t distance $r$ for both FSPM and EPM due to concentrated tangential force. The value of $\sigma_z$ first decreases sharply and then starts oscillating for FSPM as $r$ increases for both values of time where its value oscillates as $r$ increases for both values of time for EPM.
Figure 22 shows the variation of normal stress component $\sigma_{zz}$ w.r.t distance $r$ for FSPM due to concentrated normal force. The value of $\sigma_{zz}$ first decreases and then start oscillating for FSPM as $r$ increases for both values of time.

Figures 23 and 25 show the variation of pore pressure $p$ w.r.t distance $r$ for FSPM due to concentrated normal and pressure sources. The value of $p$ first increases sharply in the range $0 \leq r \leq 2$ and then oscillates for FSPM as $r$ increases for both values of time.

Figure 24 shows the variation of pore pressure $p$ w.r.t distance $r$ for FSPM due to concentrated tangential source. The value of $p$ decreases and then oscillates for FSPM as $r$ increases for both values of time.

Figure 26 shows the variation of tangential stress $\sigma_{zr}$ w.r.t distance $r$ for both FSPM and EPM due to concentrated normal force. The value of $\sigma_{zr}$ first increases sharply and then starts oscillating for time $t=0.50$ whereas for the time $t=0.75$, the value of $\sigma_{zr}$ first
increases and then converges near the boundary surface as \( r \) increases for FSPM. And for EPM it value first increases monotonically and then starts oscillating for time \( t=0.50 \) whereas for the time \( t=0.75 \) the value of \( \sigma_{zr} \) converges near the boundary surface as \( r \) increases.

Figure 27 shows the variation of tangential stress \( \sigma_{zr} \) w.r.t distance \( r \) for both FSPM and EPM due to concentrated source. The value of \( \sigma_{zr} \) decreases gradually for time \( t=0.50 \) whereas for the time \( t=0.75 \) the value of \( \sigma_{zr} \) first decreases and then converges near the boundary surface as \( r \) increases.

Figure 28 shows the variation of tangential stress \( \sigma_{zr} \) w.r.t distance \( r \) for FSPM due to concentrated pressure source. The value of \( \sigma_{zr} \) decreases gradually and then starts oscillating for time \( t=0.50 \) whereas for the time \( t=0.75 \) the value of \( \sigma_{zr} \) first decreases and then converges near the boundary surface for FSPM as \( r \) increases.

6 Conclusions

1) In the present work, we obtain the components of displacement, stress and pore pressure due to concentrated source and source over circular region in the time domain and frequency domain in saturated porous media with incompressible fluid.

2) Near the application of the source, the porosity effect decreases the values of \( \sigma_{zz} \) for normal force, tangential force and pressure source where it increases the values of \( \sigma_{zr} \) for normal force and tangential force but decreases the values for pressure source, due to concentrated source in the time domain.

3) In frequency domain, porosity effect increases the values of \( \sigma_{zz}, \sigma_{zr} \) and \( p \) for normal force, tangential
force and pressure source for source over circular region. Also away from the source the porosity effect decreases the values of \( \sigma_z \) for normal force and increases the values for tangential force and pressure source whereas it decreases the value of \( \sigma_y \) for normal force and monotonically increases for tangential force and pressure source.

4) Thus, the problem analyzed is a significant problem of continuum mechanics. The results obtained as a consequence of this research work should be beneficial for researchers working on saturated porous media with incompressible fluid. By introducing various sources, the assumed model presents a more realistic model for future study.

References


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